

Transfer Function Model Based Analysis of Permanent Magnet Synchronous Motor with Controllers

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Abstract: Permanent Magnet Synchronous Machine (PMSM) drives have been increasingly applied in a wide variety of industrial and mechatronics applications over other conventional drives. The reason comes from the advantages of PMSM: high power density and efficiency, high torque to inertia ratio, and high reliability. In this paper, the study and the design analysis of controllers and controller methods for PMSM is given. An existing BLDC motor for an electric forklift benchmarks the expected performances of the PMSM design. In the first part there is account working principle and design of PMSM. Mathematic model of PMSM is used for calculation for behavior of PMSM. Later, the mathematical modeling of PMSM has been done through Transfer Function approach. Later, obtained transfer function was subjected to controller approach which determines the performance of the PMSM. Initially, the Zeiglers-Nicholas PID controller method is used to determine the PMSM performance by two different sub controllers of ZN say, ZN-PI and ZN-PID Controllers. Next to this method, Pole Placement Technique is used for comparison of ZN-PI, PID. Characteristic Ratio Assignment (CRA) method is also used to determine the PMSM performance which was the main area of interest. This CRA method is solved with specimen problems initially and then solved for the transfer function that is derived in earlier stage. To know the performance of PMSM, CRA-PID controller is chosen in this thesis work. This method is mainly based on Pole Placement Technique. And it works on transient response of the transfer function. For the extension to this work, Fuzzy PI Logic Controller is also used to compare with CRA method. Finally, the specimen simulation problem is also performed in order to determine the speed, torque and current characteristics.

Keywords: Permanent Magnet Synchronous Motor, Charateristic Ratio Assignment method, Fully Logic controller, PID controller.

I. INTRODUCTION

Synchronous motors with a three-phase stator winding and a rotor with permanent magnets (Alternate Current –AC motors) belong to the latest generation of motors. They are applied as drives to machine tools and robots. Unlike Direct Current (DC) /brushes/ motors and Electrically Commuted (EC) /DC brushless/ motors, the Permanent Magnet Synchronous Motors (PMSM) may be configured as linear motors, which nowadays came to use in robotic applications as well. The motors work on the principle of simultaneous control of amplitude and frequency of all three terminal harmonic currents or voltages. The rotor magnetic field is supplied by permanent magnets instead of electromagnets.

The availability of these Permanent Magnets with considerable energy density leads to the development of AC motors now-a-days. These PM machines are called as compact machines which were replaced electro-magnets, which have windings and requires external electrical source. The characteristics of a permanent magnet machine are highly dependent on the rotor structure. The rotor can be implemented in various ways.

In this paper, the mathematical modeling of PMSM [1] is done and performance of PMSM is observed. With that result comparison between with and without conventional controllers and fuzzy controller were done as well.

Some specific applications of PMSM are

- In PM machines most of the losses are developed in the stator in terms of copper, eddy current and hysteresis losses. Rotor losses are negligible. Hence for a given frame size, the motor that develops lower loss will be capable of a higher power density which will helpful in robotic operation.
- PMSM has high torque to inertia ratio than induction motors which will be helpful to use these machines in Drones.[3]

1.1 Used Notations:

The model covers the relations of the current and voltage equilibrium and appropriate relations of the voltage distribution for individual phases of the three-phase system. The model contains a number of parameters. Their notation and appropriate units are given as follows:

- R_s = stator resistance[Ω, ohm]
- L_q, L_d = quadrature and direct axis inductance [H, Henry]
- Φ = rotar magnetic flux [Wb, weber]
- P = number of pole pairs
- I_q, I_d = quadrature and direct axis currents [A, Amperes]
- E_b = back emf[V, Volts]
- p = derivative with respect to time
- T_e = electromagnetic torque[N – m, newton meters]
- T = load torque[N – m, newton meters]
- J = moment of inertia[Kg – m²]
- B = friction coefficient
- ω_e = angular rotation[rad/sec]
- K_t = torque constant
- λ_d, λ_q = flux linkages[Wb, weber]
- λ_{af} = mutual flux between magnet and stator.

II. MATHEMATICAL MODELING OF PMSM

The modelling of PMSM to obtain the transfer function between output and command input which was derived as follows [2]

$$V_q = R_s i_q + p\lambda_q + \omega_e \lambda_d \tag{1}$$

$$V_d = R_s i_d + p\lambda_d - \omega_e \lambda_q \tag{2}$$

$$T_e = \left(\frac{3}{2}\right) P(\lambda_{af} i_q + (L_d - L_q) i_d i_q) \tag{3}$$

$$T_e = T_l + B\omega_r + Jp\omega_r \tag{4}$$

$$\omega_e = P\omega_r \tag{5}$$

Where $\lambda_d = L_d i_d + \lambda_{af}$ (6)

$$\lambda_q = L_q i_q \tag{7}$$

V_d, V_q are d,q axis voltages; λ_d, λ_q are d,q axis flux linkages; i_d, i_q are d,q axis currents; P is the number of pole pairs; L_q, L_d are d,q axis inductance; p is the derivative operator; T_l, T_e load torque and electric torque respectively; B is the damping coefficient; J is the moment of inertia; ω_e is the electric speed. λ_{af} is the mutual flux or airgap flux.

Assuming all the equations from (1) to (3) are non linear and i_d is forced to zero according to vector control of PMSM. Then equations (1) to (3) can be solved as given below

$$v_q = R_s i_q + L_q p i_q + \omega_e \lambda_{af} \tag{8}$$

$$v_d = -\omega_e L_q i_q \tag{9}$$

$$T_e = \left(\frac{3}{2}\right) P \lambda_{af} i_q - K_t i_q \tag{10}$$

Where K_t is torque constant, by solving all the equations the PMSM transfer function can be written as
Transfer Function of PMSM:

$$\frac{W_r(S)}{V_q(S)} = \frac{K_t}{(R + L_q S)(J_S + B) + K_t P \lambda_{af}} \tag{11}$$

2.1 Block Diagram of Permanent Magnet Synchronous Motor:

The block diagram of the permanent magnet synchronous motor is shown below.

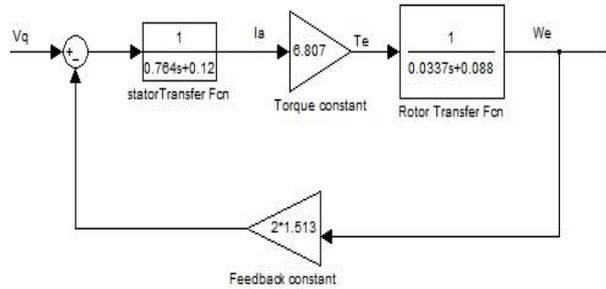


Fig:2.1 Block diagram of PMSM

The values of all parameters are taken from [2] and are given below.

- $K_t = 6.807$ [N-m/amp]
- $\lambda_{af} = 1.513$ [V/rad/sec]
- $J_S = 0.0337$ [kg-m²]
- $R = 0.12$ [ohm]
- $L_q = 0.764$ [mH]
- $B = 0.086$
- $P = 2$ (pole pairs)

Transfer Function of PMSM:

$$\frac{W_r(S)}{V_q(S)} = \frac{b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{12}$$

The overall transfer function in numerical value is written below.

$$\frac{W_r(S)}{V_q(S)} = \frac{4.705S + 2.2190}{S^3 + 7.504S^2 + 3.365S + 2.702} \tag{13}$$

III. PI CONTROLLER DESIGN

The integral action is proportional to the integral of the control error, i.e.

$$u(t) = K_i \int_0^t e(\tau) d\tau \tag{14}$$

K_i is the integral gain. It appears that the integral action is related to the past values of the control error. The value of K_i and K_p are given below

$$K_i = \frac{-w_1 \sin \theta}{|G(j\omega)|} \quad K_p = \frac{\cos \theta}{|G(j\omega)|} \tag{15}$$

The overall transfer function of PI controller is shown below $G_c(S) = K_p + \frac{K_i}{S}$ The value of $K_p = 3.1132$, $K_i = 1.5046$

PI controller T.F = $\frac{3.1132S + 1.5046}{S}$

The overall transfer function including PI controller is shown below.

$$T.F = \frac{14.647S^2 + 13.987S + 3.3387}{S^4 + 7.504S^3 + 3.36S^2 + 2.702S} \quad (16)$$

The overall block diagram of PI controlled PMSM is shown below.

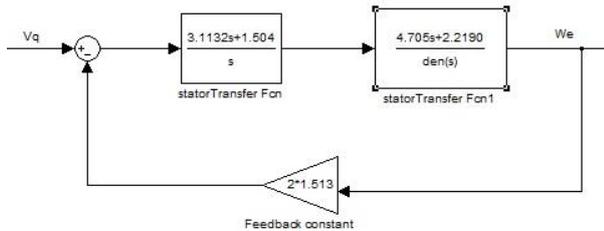


Fig 3.1: Block diagram of PMSM with PI Controller

IV. CHARACTERISTIC RATIO ASSIGNMENT METHOD

Characteristic Ratio Assignment Method is type of controller designing approach to control the transient response of a linear time invariant control systems. The transient response of a transfer function of nth order system can be controlled by ‘n-1’ parameters or characteristic ratios (α_i) where $i = n-1$ and a time constant (τ). This technique is best suitable for tuning, controller for the desired system. Characteristic polynomial coefficients and time domain responses are the main aspects that are focused in CRA method [11].

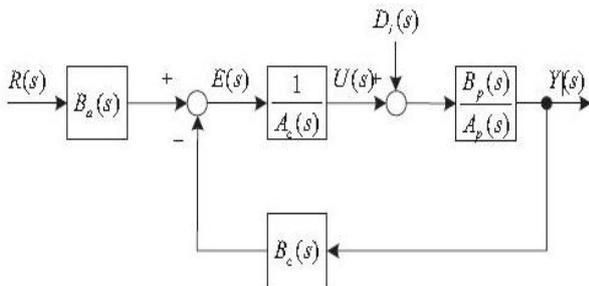


Fig4.1: Block Diagram of a Control System.

In a control system, the damping ratio can be optimized and also it relates with characteristic ratio. The block diagram of a control system is shown above. The transfer function of the system is given below

$$\frac{Y(S)}{R(S)} = \frac{B_a(S)B_p(S)}{A_c(S)A_p(S) + B_c(S)B_p(S)}$$

where $B_p(S)=K$, $B_a(S) = K_i$,
 $B_c(S) = K_d S^2 + K_p S + K_i$
 $A_p(S) = T_{12} S^2 + (T_{12} + 2T_2) S$ (plant characteristic equation),
 $A_c(S) = S$

The desired characteristic equation in generalized form is given below.

$$P(S) = T_{12} T_2 S^3 + (T_{12} + 2T_2 + K K_d) S^2 + (1 + K K_p) S + K K_i \quad (17)$$

The relation of Characteristic Equation can be written as shown below

$$P(S) = a_n S^n + a_{n-1} S^{n-1} + \dots + a_1 S + a_0, \forall a_i > 0 \quad (18)$$

where characteristic ratios can be found as follows

$$\alpha_1 = \frac{a_1}{a_0 a_2},$$

$$\alpha_2 = \frac{a_2}{a_1 a_3},$$

$$\dots$$

$$\alpha_{n-1} = \frac{a_{n-1}}{a_{n-2} a_n}$$

The inverse of characteristic equation is given as

$$b_0 = \frac{a_0}{a_1}, b_1 = \frac{a_1}{a_2}, \dots$$

The relation of coefficient ratio and characteristic pulsataces as

$$\alpha_1 = \frac{b_1}{b_2}, \alpha_2 = \frac{b_2}{b_1}, \dots, \alpha_{n-1} = \frac{b_{n-1}}{b_n}$$

The time constant is given by $\tau = \frac{a_1}{a_0}$

The characteristic equation of the PMSM using CRA is

$$S^4 + (7.504 + 4.705K_d)S^3 + (3.36 + 2.2190K_d + 4.70K_p)S^2 + (2.702 + 2.2190K_p + 4.705K_i)S + 2.2190K_i \quad (19)$$

For the above characteristic equation, desired α values are shown below which are obtained for few iterations.

$$\alpha_1 = 0.8, \quad \alpha_2 = 4.6, \quad \alpha_3 = 4.4 \quad (20)$$

with these values of α , we can find characteristic coefficients which were shown below.

$$a_0 = 2.702, \quad a_1 = 4.359, \quad a_2 = 8.7472, \quad a_3 = 3.7764, \\ a_4 = 0.3700, \quad \tau = 1.6135, \quad (21)$$

by substituting these values, in the generalized characteristic equation, the desired Characteristic Equation is obtained.

$$0.37S^4 + 3.7764S^3 + 8.7472S^2 + 4.3597S + 2.702 \quad (22)$$

from this equation we get the coefficients of PID controller as shown below.

$$K_p = 20.1 \\ K_i = 13.1 \\ K_d = 3.43$$

V. FUZZY LOGIC CONTROLLER DESIGN

Fuzzy logic controller is most commonly used technique now-a-days to develop the best control system. The advantage of this technique is, the complex design is turned to be simple. FLC is designed to perform operational laws in linguistic terms so as to eliminate all mathematical equations [7]. This method is feasible to design the operational characteristics of a system. So this method is applied to this PMSM to compare the performance of the system with other conventional controllers. The fuzzy rule base for the PMSM machine is given below.

e/Δe	NL	NM	NS	ZE	PS	PM	PL
NL	NL	NL	NLM	NM	NMS	NS	ZE
NM	NL	NLM	NM	NMS	NS	ZE	PS
NS	NLM	NM	NMS	NS	ZE	PS	PMS
ZE	NM	NMS	NS	ZE	PS	PMS	PM
PS	NMS	NS	ZE	PS	PMS	PM	PLM
PM	NS	ZE	PS	PMS	PM	PLM	PL
PL	ZE	PS	PMS	PM	PLM	PL	PL

The designing of a Fuzzy Logic Controller (using the Mamdani Fuzzy Model) requires:

1. The selection of appropriate inputs and their fuzzification.
2. The definition of the input and output membership functions.
3. The definition of the Fuzzy Rule Base.
4. The defuzzification of the output obtained after the processing of the linguistic variables with the help of a proper defuzzification technique.

Each of them has to be designed based on the result that is desired from the system.

VI. POLE PLACEMENT METHOD

Consider a linear dynamic system in the state space form In some cases one is able to achieve the goal (e.g. stabilizing the system or improving its transient response) by using the full state feedback, which represents a linear combination of the state variables, that is so that the closed-loop system, given by has the desired specifications. The main role of state feedback control is to stabilize a given system so that all closed-loop eigenvalues are placed in the left half of the complex plane. The following theorem gives a condition under which is possible to place system poles in the desired locations.

$$\begin{aligned} \dot{X} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (23)$$

In some cases one is able to achieve the goal (e.g. stabilizing the system or improving its transient response) by using the full state feedback, which represents a linear combination of the state variables, that is

$$u = -Fx$$

so that the closed-loop system, given by

$$\begin{aligned} \dot{X} &= (A-BF)x \\ y &= Cx \end{aligned} \quad (24)$$

The main role of state feedback control is to stabilize a given system so that all closed-loop eigenvalues are placed in the left half of the complex plane. The following theorem gives a condition under which is possible to place system poles in the desired locations.

Consider the nominal plant $G_0(s) = \frac{B_0(s)}{A_0(s)}$ and the

controller $C(s) = \frac{P(s)}{L(s)}$ in a simple feedback

configuration as below

$$\begin{aligned} P(s) &= p_{n_p} S^{n_p} + p_{n_p-1} S^{n_p-1} + \dots + p_0 \\ L(s) &= l_{n_l} S^{n_l} + l_{n_l-1} S^{n_l-1} + \dots + l_0 \end{aligned}$$

$$B_0(s) = b_{n_b} S^{n_b} + b_{n_b-1} S^{n_b-1} + \dots + b_0$$

$$A_0(s) = a_{n_a} S^{n_a} + a_{n_a-1} S^{n_a-1} + \dots + a_0$$

Consider a desired closed-loop polynomial

$$A_{cl}(s) = a_{n_c}^c s^{n_c} + a_{n_c-1}^c s^{n_c-1} + \dots + a_0^c$$

Design Objective

Given A0 and B0, can we find P and L such that the closed-loop characteristic polynomial is Acl(s). The characteristic equation $1 + G_0C = 0$ gives the

characteristic polynomial $A_0L + B_0P$. Does there exist P(s) and L(s) such that $A_0(s)L(s) + B_0(s)P(s) = A_{cl}(s)$

Consider the feedback loop with $G_0(s) = \frac{B_0(s)}{A_0(s)}$ and

$$C(s) = \frac{P(s)}{L(s)}$$

Assume $A_0(s)$ and $B_0(s)$ are coprime with $n = \deg A_0(s)$

Let $A_{cl}(s)$ be an arbitrary polynomial of degree $n_c = 2n - 1$

Then, there exist P(s) and L(s) with degrees $n_p = n_l = n - 1$ such that

$$A_0(s)L(s) + B_0(s)P(s) = A_{cl}(s) \quad (25)$$

The equation

$$A_0(s)L(s) + B_0(s)L(s) = A_{cl}(s)$$

is called **Diophantine** equation.

The controller is said to be bi-proper, i.e. $\deg P(S) = \tau_D = \frac{d_2}{d_1}$
 $\deg L(S)$

Usually, the polynomials $A_0(s), L(s)$ and A_{cl} are monic i.e. coefficient of highest power of S is unity. As seen before, it is very common to force the controller to contain an integrator

To achieve this, we need to rewrite the denominator of the controller as

$$L(s) = s^{-1} \bar{L}(s) \quad \bar{L}(s) = s \bar{L}(s)$$

The Diophantine equation

$$A_0(s)L(s) + B_0(s)P(s) = A_{cl}(s)$$

$$A_0(s)s\bar{L}(s) + B_0(s)P(s) = A_{cl}(s)$$

can then be rewritten as

Forcing Integration

$$\bar{A}_0(s)s\bar{L}(s) + B_0(s)P(s) = A_{cl}(s)$$

with

$$\bar{A}_0(s) = sA_0(s) \quad (26)$$

Note that because we have increased the degree of the l.h.s. by 1, now we have $\deg A_{cl} = 2n$ and need to add a closed-loop pole. Also, since $\deg L = \deg \bar{L} + 1$ to keep a bi-proper controller, we should have \deg

$$P = \deg L = \deg \bar{L} + 1 \quad (27)$$

Sometimes it is desirable to force the controller to cancel a subset of stable poles or zeros of the plant model. Say we want to cancel a process pole at $-p$, i.e. the factor $(S+P)$ in $A_0(s)$, then $P(s)$ must contain $(S+P)$ as a factor. Then the Diophantine equation has a solution if and only if $(s+p)$ is also a factor of $A_{cl}(s)$

To solve the resulting **Diophantine** equation, the factor $(s+p)$ is simply removed from both sides

Here we generalize this design.

The controller $C(s) = \frac{n_2s^2 + n_1s + n_0}{d_2s^2 + d_1s}$ and the PID

$$C_{PID}(s) = K_p + \frac{K_1}{s} + \frac{K_D s}{\tau_D s + 1} \quad \text{are equivalent when}$$

$$K_p = \frac{n_1 d_1 - n_0 d_2}{d_1^2}$$

$$K_1 = \frac{n_0}{d_1}$$

$$K_D = \frac{n_2 d_1^2 - n_1 d_1 d_2 + n_0 d_2^2}{d_1^3}$$

VII. ZIGLER'S NICHOLAS PI, PID CONTROLLER METHOD

The Ziegler-Nichols rule is a heuristic PID tuning rule that attempts to produce good values for the three PID gain parameters:

1. K_p - the controller path gain
2. T_i - the controller's integrator time constant
3. T_d - the controller's derivative time constant

Given two measured feedback loop parameters derived from measurements:

1. the period T_u of the oscillation frequency at the stability limit
2. the gain margin K_u for loop stability with the goal of achieving good regulation

The PID tuning coefficients for ZN method is given below.

$$\begin{aligned} K_p &= 0.6K_u \\ K_i &= 2K_p/T_u \\ K_d &= K_p T_u / 8 \end{aligned} \quad (28)$$

Where

$$\begin{aligned} K_u &= \frac{1}{|G(jw)|_{wcr}} \\ T_u &= \frac{2\pi}{w} \end{aligned} \quad (29)$$

Tuning rules work entirely well when you have a simple controller, a framework that is straight, monotonic, and slow, and a reaction that is commanded by a solitary shaft exponential "slack" or something that demonstrates a great deal like one.

Real plants are probably not going to have a flawless first-arrange slack trademark, however this estimation is sensible to depict the recurrence reaction rolloff in a dominant part of cases. Higher-arrange shafts will present an additional stage move, be that as it may. Regardless of the possibility that they don't influence the state of the pick up rolloff much, the stage move matters a ton to circle dependability. You can't rely on a solitary "slack" post to coordinate both the plentifulness rolloff and the stage move precisely.

So the Ziegler-Nichols display presumes an extra anecdotal stage conformity that does not mutilate the expected greatness rolloff. At the strength edge, there is a 180 degree stage move around the criticism circle (Nyquist's security rule). A first request slack can contribute close to 90 degrees of that stage move.

Whatever remains of the watched stage move must be secured by the manufactured stage change. The stage modification is dared to be a straight line somewhere around zero and the basic recurrence where 180 degrees of stage move happens. A "straight line" stage move relates to an immaculate time delay.

VIII. SIMULATION AND RESULTS

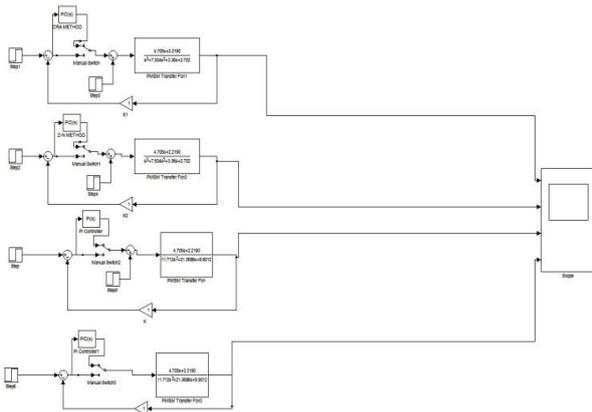


Fig 8.1 Matlab simulink block diagram for ZN (PI, PID), CRA and Pole Placement.

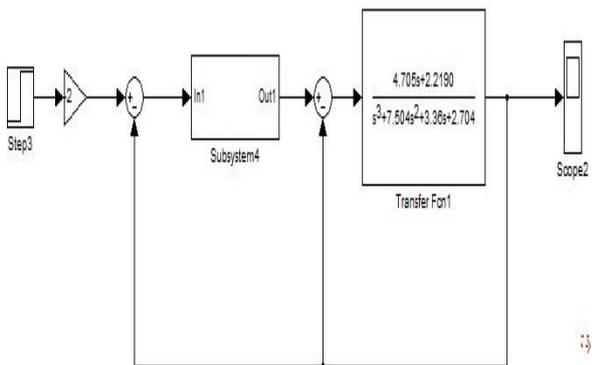


Fig 8.2: Simulink block diagram for Fuzzy Logic Controller

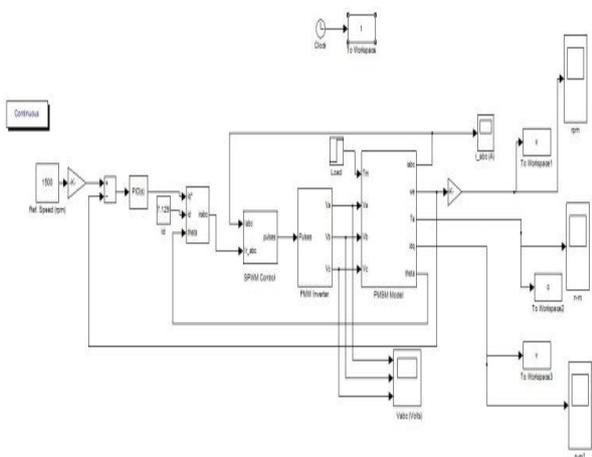


Fig 8.2 Simulink Model of PMSM

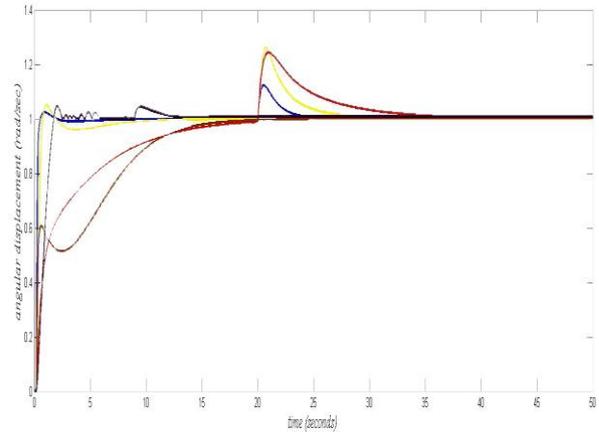


Fig 8.3 Comparison of Simulations

IX. CONCLUSION

In this Paper, PMSM performance is compared with few conventional controllers. In addition to that CRA method is introduced to determine the performance of PMSM, which was also compared with other controllers. CRA (Characteristic Ratio Assignment) method is based on the transient response analysis, from which the formulae for calculating PI/PID controller parameters are derived. These formulae helped mainly in PMSM (Permanent Magnet Synchronous Motor) performance that can be observed from the process step response. This performance was compared with other conventional controllers like ZN-PI, ZN-PID, Pole Placement Method, Fuzzy Logic Controller. These results are in quite simple and straightforward time-domain tuning approach. Simulations are performed on the different processes with Time Delay Systems have shown that the proposed method (CRA) gives better results. From the simulation results it is observed that the controller is effectively rejecting the disturbances. Therefore the CRA method results in fast and stable closed-loop responses. The drawbacks of this approach are that the method requires a stable open-loop process response to determine the appropriate controller parameters.

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